

# Galactic dynamo seeds from non-superconducting spin-polarised strings

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## Abstract

Earlier Enqvist and Olesen have shown that formation of ferromagnetic planar walls in vacuum at GUT scales in comoving plasmas may generate a large scale magnetic field of  $B_{now} \simeq 10^{-14}G$ . In this paper we show that starting from classical Einstein-Cartan-Maxwell strong gravity, a spin-polarised ferromagnetic cylinder gives rise to a cosmological magnetic field of the order  $B_{now} \simeq 10^{-22}G$ . Vorticity of cylinder is used to obtain galactic magnetic fields. Magnetic fields up to  $B \sim 10^9G$  can be obtained from the spin density of the cylinder. If matching conditions are used cosmological magnetic fields of the order of  $B \sim 10^{-16}R \frac{Gauss}{cm}$  where  $R$  is the radius of the cosmic strings. For a cosmic string with the radius of an hydrogen atom the cosmic magnetic field is  $B \sim 10^{-32}Gauss$  which is enough to seed galactic dynamos. Current of these cosmic strings are computed and its shown that these strings are non-conducting since the electric current is much weaker than the one produced by string dynamos. Taking into account vorticity of the cosmic string one obtains a B-field which coincides with the IGM field of  $10^{-5}Gauss$ .

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# 1 Introduction

Earlier the author has published a paper in CQG [1] where spin-polarised cylinders with magnetic fields were used as a tool to test Einstein-Cartan theory of gravitation [2]. In the present paper we compute from the negative pressure exact solutions of ECM equations two expressions one for the magnetic field in terms of torsion which we show to yield a B-field of  $10^{-16}G$  compatible with the values estimated by Barrow et al [3] using only general relativity plus usual Maxwell equations, and the other of the magnetic field in terms of the vorticity of the spin-polarised cylinder which also produces the  $\mu G$  galactic magnetic fields as seen for example in the Milky way. Both results shows that ECM gravity is compatible with the observed astronomical data of dynamo mechanism not only for galactic dynamos but also for the cosmological magnetic fields as seeds for these galactic dynamos. This shows also that the recently present author efforts [4] to show that torsion of space-time could be compatible with dynamo mechanism and primordial magnetic fields. When matching conditions are used on spin-polarised non-rotating cylinders one obtains a relation between the large scale magnetic fields and the radius of the cosmic strings. For strings of radius of scale of hydrogen atom B-fields as low as  $10^{-24}Gauss$  are found. In section 2 we discuss the magnetogenesis of the cosmic string in terms of the matching conditions in Riemann-Cartan spacetime. In section 3 instead of this vorticity free case we consider the Harrison-Rees vorticity and compute the magnetic field from the spin-polarised cosmic string which yields the IGM of  $10^{-5}Gauss$ .

## 2 Matching conditions for cosmic strings in torsioned spacetime and magnetogenesis

Recently we have shown [5] that Soleng [6] cylinder geometry given by

$$ds^2 = -(e^\alpha dt + M d\phi)^2 + r^2 e^{-2\alpha} d\phi^2 + e^{2\beta-2\alpha} (dr^2 + dz^2). \quad (1)$$

in the particular case of  $\alpha = \beta = 0$  can be shown to be a solution of Einstein-Cartan-Maxwell (ECM) field equations which represents a magnetized spin polarised cylinder in torsioned spacetime where the RC rotation vanishes when one applies the matching condition in this non-Riemannian space.  $M$

is a function of the radial coordinate  $r$ . Exterior solution is the same as used by Soleng in the case of thick spinning cosmic strings [6] and represents an exterior solution of Einstein's vacuum field equation

$$R_{ik} = 0 \quad (2)$$

( $i, j = 0, 1, 2, 3$ ) as

$$ds^2 = -dt^2 - 2adtd\phi + dr^2 + (B^2(r + r_0)^2 - a^2)d\phi^2 + dz^2 \quad (3)$$

Here  $a$ ,  $B$  and  $r_0$  are constants. Before proceed in this analysis let us consider the above metric (1) in terms of the differential one-form basis

$$\theta^0 = e^\alpha dt + M d\phi, \quad (4)$$

$$\theta^1 = e^{\beta-\alpha} dr, \quad (5)$$

$$\theta^2 = r e^{-\alpha} d\phi, \quad (6)$$

$$\theta^3 = e^{\beta-\alpha} dz. \quad (7)$$

Polarisation along the axis of symmetry is considered and the Cartan torsion is given in terms of differential forms by

$$T^i = 2k\sigma\delta_0^i\theta^1\wedge\theta^2 \quad (8)$$

where  $\sigma$  is a constant spin density. For computational convenience we adopt Soleng's definition [6] for the RC rotation  $\Omega$

$$\Omega := -\frac{1}{2}\sigma + \frac{M'}{2r} \quad (9)$$

where  $\Omega$  is the cylinder RC vorticity. Cartan's first structure equation is

$$T^i = d\theta^i + \omega^i_k \wedge \theta^k \quad (10)$$

and determines the connection forms  $\omega^i_j$ . The connection one-forms are given by

$$\omega_1^0 = -\Omega\omega^2 \quad (11)$$

$$\omega_2^0 = \Omega\omega^1 \quad (12)$$

$$\omega_3^0 = 0 \quad (13)$$

$$\omega_2^1 = -\Omega\omega^0 - \left(\frac{1}{r}\right)\omega^2 \quad (14)$$

while others vanish. From the Cartan's second structure equation

$$R^i_j = d\omega^i_j + \omega^i_k \wedge \omega^k_j \quad (15)$$

where the curvature RC forms  $R^i_j = R^i_{jkl}\theta^k \wedge \theta^l$  where  $R^i_{jkl}$  is the RC curvature tensor. This is accomplished by computing the RC curvature components from the Cartan structure equations as

$$R_{0101} = \Omega^2, \quad (16)$$

$$R_{0112} = \Omega', \quad (17)$$

$$R_{0202} = \Omega^2, \quad (18)$$

$$R_{1201} = \Omega', \quad (19)$$

$$R_{1212} = 3\Omega^2 - 2\Omega\sigma, \quad (20)$$

others zero. The dash here represents the derivative *w.r.t* to the radial coordinate  $r$ . From the curvature expressions above it is possible to built the ECM field equations as

$$G^i_k = kT^i_k \quad (21)$$

where  $G^i_k$  is the Einstein-Cartan tensor and  $T^i_k$  is the total energy-momentum tensor composed of the fluid tensor  $T^i_k = (\rho, p_r, 0, p_z)$  and the electromagnetic field tensor

$$t^i_k = (F^i_l F^l_k - \frac{1}{2}\delta^i_k (E^2 - B^2)) \quad (22)$$

where  $F_{0\gamma}$  correspond to the electric field  $\vec{E}$  while  $F_{\alpha\beta}$  components of the Maxwell tensor field  $F_{ij}$  correspond to the magnetic field  $\vec{B}$ . Here we consider that the electric field vanishes along the cylinder, and  $\alpha = 1, 2, 3$ . Thus the natural notation  $E^2 = (\vec{E})^2$  and the same is valid for the magnetic field. Thus explicitly the ECM equations read

$$-3\Omega^2 - \sigma\Omega = -k\left(\rho + \frac{B_z}{2}\right) \quad (23)$$

$$\Omega^2 = k\left(p_r - \frac{B_z}{2}\right) \quad (24)$$

$$\Omega' = 0 \quad (25)$$

$$\Omega^2 + \sigma\Omega = -k(p_z + \frac{B_z}{2}) \quad (26)$$

Note that equation (25) is the simplest to solve and yields  $\Omega = \Omega_0 = \text{constant}$ . This actually from the Riemann curvature expressions above shows that the Riemann tensor vanishes outside the cylinder shell string which shows that from the metrical point of view the spacetime is flat or Minkowskian. Therefore so far just from the ECM field equations we cannot say that the cylinder is static. Before proceed therefore it is useful to show that this results from the Arkuszewski-Kopczynski-Ponomarev (AKP) [7] junction conditions for Einstein-Cartan gravity which match an interior solution of ECM field equations to the exterior vacuum solution given by the geometry given by expression (3). The AKP conditions are

$$g_{ij,r}|_+ = g_{ij,r}|_- - 2K_{r(ij)} \quad (27)$$

for  $(i, j)$  distinct from the  $r$  coordinate, where the contortion tensor is

$$K_{ijk} = \frac{1}{2}(T_{jik} + T_{jki} - T_{ijk}) \quad (28)$$

where  $T_{jik}$  is the Cartan torsion. The plus and minus signs here correspond respectively to the exterior and interior spacetimes respectively. The others AKP conditions state that the fluid elements do not move across the junction surface, the stress normal to the junction surface vanishes and that

$$g_{ij}|_+ = g_{ij}|_- \quad (29)$$

which is the general relativistic Lichnerowicz condition. From the cylinder geometry one obtains

$$g_{t\phi}|_+ = g_{t\phi}|_- \quad (30)$$

$$g_{\phi\phi}|_+ = g_{\phi\phi}|_- \quad (31)$$

$$g_{t\phi,r}|_+ = g_{t\phi,r}|_- - T_{tr\phi} \quad (32)$$

$$g_{\phi\phi,r}|_+ = g_{\phi\phi,r}|_- - 2T_{\phi r\phi} \quad (33)$$

which for the exrerior and interior of the cylinder matching at  $r = R$  one obtains

$$a = M(R) \quad (34)$$

$$B^2(R + r_0)^2 = R^2 - M^2 \quad (35)$$

$$\Omega_0 = -\frac{1}{2}\sigma_0 + \frac{M'}{2R} \quad (36)$$

$$0 = \sigma_0 R - M' \quad (37)$$

$$B^2(R + r_0) = R - MM' + MR\sigma_0 \quad (38)$$

Substitution of  $M'$  above into expression (36) yields the desired result that the Riemann-Cartan rotation  $\Omega$  vanishes. The remaining junction conditions [7] yield

$$B^2 = \frac{R}{(R + r_0)} \quad (39)$$

$$\sigma_0 = \frac{4}{R}\left[1 - \frac{R + r_0}{R}\right] \quad (40)$$

Substitution of these results into the exterior metric yields the following exterior spacetime for the spin polarised cylinder

$$ds^2 = -dt^2 - 2\sigma_0 R^2 dt d\phi + dr^2 + R\left(\frac{(r + r_0)^2}{R + r_0} - \sigma_0^2 R^3\right)d\phi^2 + dz^2 \quad (41)$$

Now going back to the ECM equations we obtain the following constraints

$$\rho = \frac{B_z}{2} \quad (42)$$

which states that the energy density is purely of magnetic origin. Besides to keep the stability of the spin polarised cylinder and its static nature one obtains that the radial pressure  $p_r > 0$  while the axial pressure is negative or  $p_z < 0$ . Indeed from the field equations we obtain  $p_z = -\frac{B_z^2}{2}$  while  $p_r = \frac{B_z^2}{2}$ . Physically this is in accordance with the fact that the radial stresses, here including the magnetic stress ( $T_1^1$ ) must vanish at the cylinder surface. The heat flow also vanishes which in the ECM field equations implies that the RC rotation must be in principle constant. Note that the signs of the pressures along orthogonal directions indicate that the magnetic string undergoes gravitational collapse which in turn helps dynamo mechanism. Physical applications of the model discuss here may be in the investigation of the gravitational extra effects on the well-known Einstein-de Haas effect due

to the non-Riemannian effects from the spin density. To use this Einstein-de Haas idea we may simply cancel the term in front of  $d\phi^2$ . This yields

$$B^2 = \sigma_0^2 R^4 \quad (43)$$

whose square root yields

$$B = \pm \sigma_0 R^2 \quad (44)$$

Note that when the string radius is the radius of an H atom,  $R_H \sim 10^{-8}cm$  this yields

$$B = \frac{c^3 T}{8\pi G} R^2 \quad (45)$$

when  $G = G_f \sim 10^{30} cgsunits$  is the f-meson gravity dominance gravitational constant, this yields

$$B = 10^{-16} \times R^2 \quad (46)$$

which in turn yields

$$B = 10^{-32} Gauss \quad (47)$$

In the case of superconducting cosmic strings [8, 9] the relation between electric current  $J$  and magnetic field is given by the Biot-Savart law for the magnetic cylinder

$$B = \frac{2J}{R} \quad (48)$$

which immediately tells us that the electric current over the string is extremely small. As shown by Witten [9] in general superconducting cosmic strings possesses strong electric currents and not weak like that. Note that earlier D Battefeld et al [10] have shown that by investigating the magnetogenesis and show that  $B \sim 10^{-29}G$  has been obtained on  $5 \rightarrow 5kpc$  are needed to account for magnetic fields in spirals galaxies, accounts using cosmic strings; during the collapse of protogalactic clouds. This value is only 3 orders of magnitude we obtained here. Actually gravitational waves and dragging effects leads to  $10^{-28}Gauss$ .

### 3 Spin-torsion polarised cosmic strings and large scale magnetic fields

In this section we shall consider a rotating string instead of non-rotating string of the last section. Let us now consider the spin-polarised cylinder

metric

$$ds^2 = -[exp(\alpha)dt + Md\phi]^2 + r^2 exp(-\alpha)d\phi^2 + exp(2\beta - 2\alpha)[dr^2 + dz^2] \quad (49)$$

as given in Soleng [4]. By making use of ECM equations this reduces to

$$\frac{kB_z^2}{2} = \Omega^2 \quad (50)$$

where  $\Omega^2$  is the square of vorticity of the spin-polarised cylinder. The remaining ECM equations are

$$k\rho - \frac{kB_z^2}{2} = \Omega^2 - 2\sigma\Omega \quad (51)$$

$$kp_z + \frac{kB_z^2}{2} = -\Omega^2 + 2\sigma\Omega \quad (52)$$

Here  $\sigma$  is the spin-density. Let us now express the cylindrical string as

$$ds^2 = -[dt + k(k\sigma^2 - \frac{B_z^2}{\sqrt{2}})r^2d\phi^2]^2 + r^2d\phi^2 + dr^2 + dz^2 \quad (53)$$

Thus magnetic energy to  $B_z^2$  is proportional to spin density  $\sigma^2$ . Thus one obtains

$$B_z \sim \sigma \quad (54)$$

This formula is obtained by the cancelment of the term between spin-torsion effects and magnetic fields as in Einstein-de Haas effect. Since the torsion scalar  $T$  is given by

$$T = \frac{4\pi G}{c^3}\sigma \quad (55)$$

Now let us show the physical importance of this correction by showing that inversion of this last expression which yields

$$\sigma = \frac{c^3}{4\pi G}T \quad (56)$$

and substitution in the now corrected expression (2) yields an important expression between the magnetic field and torsion already discussed in previously to estimate magnetic fields obtained from torsion fields [?]. Thus

$$B_z \sim \frac{c^3}{4\pi G}T \quad (57)$$



By making use of the torsion field on earth laboratory as given in Hughes-Drever experiment by Laemmerzahl [5] as  $T \sim 10^{-17}cm^{-1}$ , and the strong gravity gravitational constant [2]  $G_f = 10^{30}cgsunits$  one obtains the following magnetic field  $B \sim 10^{-22}Gauss$  which as we wish to prove very well within the magnetic field to seed galactic dynamos as shown recently by Barrow et al [13]. Let us now consider the cosmological magnetic field obtained from the ECM equations in terms of the rotation the first ECM equations is

$$B_z = 2\frac{1}{\sqrt{k}}\Omega \quad (58)$$

where  $\Omega$  is the Harrison-Rees vorticity which for galaxies is  $10^{-9}rads^{-1}$ . From the above value for Einsteins gravitational constant one obtains  $B_z \sim 10^{-5}Gauss$  which is the intergalactic magnetic field. Note that investigation of magnetogenesis [11] in Einstein-Cartan gravity is not a novel subject but in the previous works [12] much more attention has been given to torsion than to metric effects like in this paper. Note that cosmic strings we have addressed here are non-superconducting contrary to ones investigated by Witten and more recently by Dimopoulos and Davies [14] with friction and backreaction.

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## References

- [1] L.C.Garcia de Andrade, Class. Quantum Gravity (2001) 18,3097.
- [2] M.L.Bedran and L.C.Garcia de Andrade, Prog Theor Phys (1983) 12, 1583.
- [3] H.Soleng, Class. and Quant. Gravity 7,(1990),999.
- [4] C.Lämmerzahl, Phys.Lett. A 228 (1997) 223.
- [5] L C Garcia de Andrade, Gen. Rel. and Gravitation 35 (2003),1279.
- [6] H. Soleng, Gen. Rel. and Gravitation (1992) 24,1,111.
- [7] W. Arkuszewski, W. Kopczynski and V.N. Ponomarev, Comm. Math. Phys.(1975)45,183. A. Beessange, Class. and Quantum Gravity (2000) 17,2509.
- [8] Li-Shing Hou and Wei-tou Ni, Rotatable torsion balance equivalence principle experiment for spin-polarised, Mod. Phys. Lett. A (2000).
- [9] E. Witten, Nucl Phys **B** 249 (1985) 557.
- [10] D Battefeld, T Battefeld, D Wesley and M Wyman, JCAP 0802: 001 (2008).
- [11] L Widrow, Rev Mod Phys **74**: 775 (2001). M Turner and L Widrow, Phys Rev **D** (1988). T. Prokopec, O Tornkvist and R Woodward, Phys Rev Lett. 89: 101301, (2002). A Ruzmakin, D D Sokoloff, and A. Shukurov, Magnetic fields in Galaxies, Kluwer (1988). L C Garcia de Andrade, Nuclear Phys **B** (2011). L Garcia de Andrade, Phys Lett **B** 468, 28 (2011). B. Ratra, Caltech preprint. L Garcia de Andrade, Lorentz violation bounds from torsion trace and radio galactic dynamos, Phys Rev D (Brief Reports) (2011).
- [12] L C Garcia de Andrade, Nuclear Phys **B** (2011). L Garcia de Andrade, Phys Lett **B** 468, 28 (2011). L Garcia de Andrade, Lorentz violation bounds from torsion trace and radio galactic dynamos, Phys Rev D (Brief Reports) (2011). L Garcia de Andrade, Galactic dynamo seeds

from Einstein-Cartan gravity and f-meson dominance gravity, *Advances studies in theoretical physics* (2013) in press.

[13] J D Barrow, C Tsagas and K Yamamoto, *Phys Rev D* 86, 023553 (2012).

[14] K Dimopoulos and A C Davis, *Phys Rev D* 57: 692 (1998).